4.3 pg.175 # 13, 15, 19, 21, 23, 25

13. $y = \sec^{-1}(2s+1) = \dfrac{1}{|2s+1|\sqrt{(2s+1)^2-1}} \cdot 2$

$= \dfrac{2}{|2s+1|\sqrt{4s^2+4s}} = \dfrac{1}{|2s+1|\sqrt{s^2+s}}$

15. Note: If $y = \csc^{-1}x$, then $\dfrac{dy}{dx} = \dfrac{-1}{|x|\sqrt{x^2-1}}$

$y = \csc^{-1}(x^2+1)$

$\dfrac{dy}{dx} = \dfrac{-1}{|x^2+1|\sqrt{(x^2+1)^2-1}} \cdot 2x = \dfrac{-2x}{(x^2+1)\sqrt{x^4+2x^2}}$

$= \dfrac{-2}{(x^2+1)\sqrt{x^2+2}}$

19. $y = \cot^{-1}(\sqrt{t-1})$

Note: If $y = \cot^{-1}(x)$, then $y' = \dfrac{-1}{x^2+1}$

$y = \cot^{-1}(\sqrt{t-1})$

$\dfrac{dy}{dt} = \dfrac{-1}{(\sqrt{t-1})^2+1} \cdot \frac{1}{2}(t-1)^{-\frac{1}{2}} = \dfrac{-1}{2t\sqrt{t-1}}$

21. $y = \tan^{-1}\sqrt{x^2-1} + \csc^{-1}x \qquad x > 1$

$$\frac{dy}{dx} = \frac{1}{1+\left(\sqrt{x^2-1}\right)^2} \cdot \frac{1}{2}(x^2-1)^{-\frac{1}{2}}(2x) + \frac{-1}{|x|\sqrt{x^2-1}}$$

$$= \frac{1}{x^2} \cdot \frac{x}{\sqrt{x^2-1}} + \frac{-1}{|x|\sqrt{x^2-1}}$$

$$= \frac{1}{x\sqrt{x^2-1}} - \frac{1}{x\sqrt{x^2-1}}$$

$$= 0$$

23. $y = \sec^{-1}(x) \qquad x = 2 \qquad y(2) = \sec^{-1}(2) = \frac{\pi}{3}$

$$\frac{dy}{dx} = \frac{1}{|x|\sqrt{x^2-1}} \qquad \text{at } x=2 \quad \frac{dy}{dx} = \frac{1}{2\sqrt{4-1}} = \frac{\sqrt{3}}{6}$$

Tangent line equation $y - \frac{\pi}{3} = \frac{\sqrt{3}}{6}(x-2)$

25. $y = \sin^{-1}\left(\frac{x}{4}\right) \qquad x = 3 \qquad y(3) = \sin^{-1}\left(\frac{3}{4}\right) \approx 0.848$

$$\frac{dy}{dx} = \frac{\frac{1}{4}}{\sqrt{1-\frac{x^2}{16}}} = \frac{1}{4\sqrt{\frac{16-x^2}{16}}} = \frac{1}{\sqrt{16-x^2}}$$

$$\text{at } x=3 \quad \frac{dy}{dx} = \frac{1}{\sqrt{16-9}} = \frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

tangent line equation $y - 0.848 = \frac{\sqrt{7}}{7}(x-3)$